РОЛЬ ГЕОМЕТРИИ В СОВРЕМЕННОЙ ФИЗИКЕ

С.Г Рубин

В работе обсуждается роль дополнительных измерений. Демонстрируется, что идея дополнительного пространства в комбинации с теорией гравитации с высшими производными является подходящим инструментом для решения фундаментальных проблем современной физики.

Ключевые слова: Дополнительное пространство, размерность, гравитация, тонкая настройка.

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ON CONFORMAL KILLING AND HARMONIC FORMS ON RIEMANNIAN SYMMETRIC SPACES

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We prove that there are no non-zero closed and coclosed conformal Killing L^2-forms on a complete, simply connected and irreducible symmetric space with negative scalar curvature. We also prove that there are no non-zero harmonic forms on a complete, simply connected and irreducible symmetric space with positive scalar curvature.

Keywords: Riemannian symmetric space, harmonic forms, closed and coclosed conformal Killing forms.

Conformal Killing forms have been defined on Riemannian manifolds more than forty-five years ago by Tachibana (see [1]) as a natural generalization of conformal Killing vector fields. Surveys of the publications on these forms can be found in the introduction to our last paper [2].

A Riemannian globally symmetric space of *non-compact type* (M,g) is complete and also (M,g) has a nonpositive sectional curvature. We also know that a Riemannian symmetric space has nonpositive (resp. non-negative) curvature operator if and only if it has nonpositive (resp. non-negative) sectional curvature (see [3]). Note that symmetric spaces of non-compact type are non-compact. After the above remarks, the assertion of the following theorem becomes obvious.

Theorem 1. *Let (M,g) be an n-dimensional $(n \geq 3)$ complete, simply connected symmetric space. If (M,g) is irreducible and its scalar curvature is negative, then there are no non-zero closed and coclosed conformal Killing L^2-forms of degree p $(1 \leq p \leq n-1)$ on (M,g).*

Theorem 2. *Let (M,g) be an 2p-dimensional $(p \geq 1)$ complete, simply connected symmetric space. If (M,g) is irreducible and its scalar curvature is negative, then there are no non-zero conformal Killing L^2-forms of degree p on (M,g).*

It is well known that a Riemannian globally symmetric space of *compact type* (M,g) is compact and also (M,g) has a nonpositive sectional curvature (see [3]). Then the following theorem holds.

Theorem 3. *Let (M, g) be an n-dimensional $(n \geq 2)$ complete, simply connected symmetric space. If (M, g) is irreducible and its scalar curvature is positive, then there are no non-zero harmonic p-forms $(1 \leq p \leq n-1)$ on (M, g).*

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О КОНФОРМНО КИЛЛИНГОВЫХ И ГАРМОНИЧЕСКИХ ФОРМАХ НА РИМАНОВЫХ СИММЕТРИЧЕСКИХ ПРОСТРАНСТВАХ

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Мы доказываем, что не существует ненулевых замкнутых и козамкнутых конформно киллинговых L^2-форм на полном односвязном неприводимом римановом симметрическом многообразии с отрицательной скалярной кривизной. Также доказываем, что не существует гармонических L^2-форм на полном односвязном неприводимом римановом симметрическом многообразии с положительной скалярной кривизной.

Ключевые слова: Риманово симметрическое пространство, гармонические формы, замкнутые и козамкнутые конформно киллинговые формы.

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ON INFINITESIMAL HARMONIC TRANSFORMATIONS ON COMPLETE RIEMANNIAN MANIFOLDS

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We prove that every infinitesimal harmonic transformation with finite kinetic energy on a complete Riemannian manifold with non-positive Ricci curvature is parallel. We apply this fact to the Ricci soliton theory.

Keywords: Complete Riemannian manifold, infinitesimal harmonic transformation, Ricci soliton.

A vector field ξ on a complete Riemannian manifold (M, g) is called an infinitesimal harmonic transformation if ξ generates a flow which is a local one-parameter group of harmonic transformations [1].

The kinetic energy of the flow (see [2, p. 2]) on a complete Riemannian manifold (M, g) generated by a complete vector field ξ is determined by the following equation $E(\xi) = \frac{1}{2} \int_M \|\xi\|^2 dVol_g$. Using the above definition we can prove the following theorem.